Collective Transports in Two Elastically Coupled Inertia Ratchets

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We report the existence of varieties of collective dynamics ranging from on-off intermittent (OOI) synchronization to fully synchronized transport in two elastically coupled inertia ratchets; and show a connection between OOI and stepwise-sliding dynamics typical of actin-myosin motors in muscles. The OOI collective transport appears as a stable transition state to fully synchronized dynamics; achieved via stepwise-sliding and accompanied by boundary crisis. We demonstrate a strong dependence of the transport on the coupling strength ϵ, and show that enhanced transports could occur in both directions as ϵ is varied.

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Transport phenomena and particularly, directed transports occur in many natural situations ranging from physical systems to chemical and biological systems. Recent research interest in transport problems is related to ratchet physics where unbiased, noise-induced transport emerges away from thermal equilibrium as a result of the action of Brownian motors [1–3]. Brownian motors, especially "ratchet" models, have been widely investigated partly due to the challenge to describe and control mechanisms of fundamental biological processes at both the cell level (e.g. transport in ion channels) and body level (muscle operations) [4]. Another source of motivation is derived from recent advances in technology wherein devices for guiding tiny particles on nano/micro scales are sought for; these include particle separation techniques, smoothing of atomic surfaces during electromigration and control of the motion of vortices in superconductors [2, 5]. Remarkably, experimental realizations of some of these practical systems have been reported [6–8].

Two basic types of ratchet models have been proposed and studied extensively. In the first class of ratchets, the particles are subject to an external unbiased driving force or additive coloured noise. These are called the rocking ratchets or correlation ratchets. The second class is the flashing ratchets, wherein the particles are driven by a flashing potential. The vast majority of these models deals with overdamped cases where noise plays a vital role in the transport process. However, recent studies have shown that the role of noise can be replaced by deterministic chaos induced by the inertial term [9]. In such inertial ratchets, the issue of current reversal was intuitively addressed [10] and later carefully reformulated [11].

In the description of the models and the characterization of transport properties, attention has been paid mostly to single particle ratchets. However, in reality one cannot find an isolated single particle system. For instance, molecular motors do not operate as a single particle but in groups that form what is known as multimotor - the most prominent example being the actin-myosin system in muscles [12]. For this reason, the relevance of many interacting particles and possible effects of collective behaviour, e.g. transport enhancement, current inversion, clustering and spontaneous current among others have been considered in the overdamped case ([13] and references therein) and the underdamped case [14–16]. Besides a few studies [14–16] on synchronization behavior and possible connections to transport properties in the underdamped cases, a large number of studies investigates the transport properties based on the coupled overdamped case.

Notably, the interaction between the coupled ratchet models described in [14], were provided through the velocity coordinates, while in [15], external forces arising from systematically designed nonlinear control were employed. Indeed, the type and topology of the coupling may differ and the specific choice of the coupling scheme will determine exclusively the system's dynamics and hence the transport property. In this letter, we report un-usual collective dynamics associated with a transition from OOI synchrony to full synchrony in a system of coupled inertia ratchets and establish a connection between on-off intermittent (OOI) synchrony and stepwise-sliding motion, typically observed in molecular motors (e.g. the actin-myosin system). We further show that the sequence of the transition from OOI to full synchrony leads to en-

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hanced transport.

In our model, a rocking ratchet potential is symmetrically perturbed by means of an elastic coupling that is provided by the displacement coordinates. The motion of two ratchets (in dimensionless form) is described by

$$\ddot{x}_i + b\dot{x}_i + \frac{dV(x_i)}{dx_i} = a\cos(\omega t) \quad (i = 1, 2), \qquad (1)$$

where the normalized time t is taken in units of the small resonant frequency ω_0^{-1} of the system; a, ω, and b are respectively the amplitude of the forcing strength, the frequency of the external forcing and the damping parameter. $V(x_{1,2})$ is the perturbed two-dimensional ratchet potential given as:

$$V(x_{1,2}) = 2C - \frac{1}{4\pi\delta}\left[\phi(x_1) + \phi(x_2)\right] + \frac{\epsilon}{2}(x_1 - x_2)^2, \quad (2)$$

where $\phi(x_{1,2}) = \sin 2\pi(x_{1,2} - x_0) + \frac{1}{4}\sin 4\pi(x_{1,2} - x_0)$; the last term is the coupling term and ϵ is the coupling strength which determines the dynamics and hence the transport properties of the Eq. (1). The parameter x_0 in $\phi(x_{1,2})$ is appropriately chosen such that the minima of $V(x_{1,2})$ are located at the integers; whereas the other parameters are taken as $\delta = 1.614324$ and $C = 0.0173$.

The perturbed 2-dimensional ratchet potential (2) for four different values of the coupling strength, $\epsilon = 0, 0.05, 0.15, 1.0$, are shown in Fig. 1. The minima and maxima of the potential are marked with blue and red color respectively. It is to be noted that as the coupling strength is increased progressively, the heights of the 2-dimensional potential $V(x_{1,2})$ decreases along the diagonal. Thus, for sufficiently large ϵ, the particles can tunnel through the barrier and move collectively in a synchronized manner as a single large entity with enhanced transport. This motion significantly determines the global dynamics and the transport properties of the coupled inertia ratchets (1).

Now, we explore this collective dynamics of the coupled ratchets (1) as ϵ is increased. At first, we consider the average bare energies [17] represented as,

$$h_{1,2} = \frac{1}{T}\int_0^T E_{1,2}(t)dt; \quad E_{1,2}(t) = \frac{p_{1,2}^2}{2} + V(x_{1,2}), \quad (3)$$

where $p_{1,2}$ is the associated momentum and $V(x_{1,2})$ is the potential. We have fixed the parameters as $a = 0.0809472, b = 0.1, x_0 = 0.82$, and $\omega = 0.67$ for which the uncoupled individual systems exhibit intermittent chaotic dynamics and left sufficient initial transients in all our simulations. $h_{1,2}$ are plotted in Fig. 2(a) as a function of ϵ in the range $\epsilon \in (0, 1)$. Above the threshold value $\epsilon > \epsilon_{th} = 0.68$, the average bare energies become identical and hence full synchrony is achieved. Just after this point, the average error state η given by

$$\eta = \frac{1}{T}\int_0^T \left[\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]\, dt, \qquad (4)$$

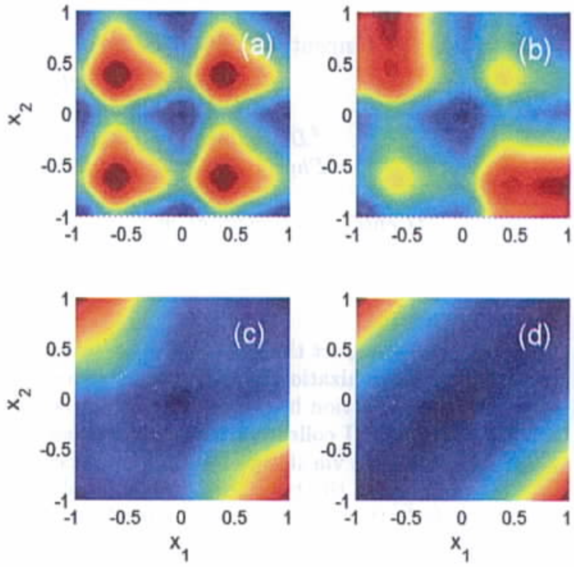

FIG. 1: Two dimensional ratchet potential $V(x_{1,2})$ for different coupling strength. The parameter $x_0 = 0.82$. (a) No interaction, $\epsilon = 0$, (b) weak coupling, $\epsilon = 0.05$, (c) moderate coupling, $\epsilon = 0.15$ and (d) strong coupling, $\epsilon = 1.0$.

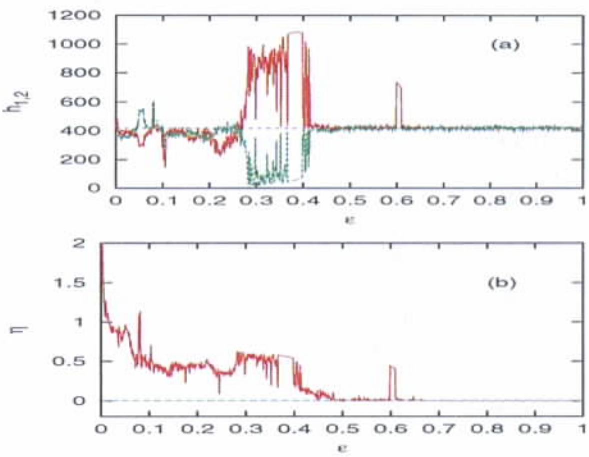

FIG. 2: Transition to collective dynamics. (a) The average energies $h_{1,2}$ vs ϵ shows the transition between intermittent and full synchronization states and (b) show the avarage error dynamics, η in the same range of ϵ. The parameters are set as: $a = 0.0809472, b = 0.1, x_0 = 0.82$, and $\omega = 0.67$.

also approaches zero as can be seen in Fig.2(b).

It is important to note that in the fully synchronized state, collective transport can occur in both the positive and negative directions, while the dynamics of each particle $x_{1,2}$ could be chaotic or absolutely periodic depending on ϵ. With moderate coupling in the range $\epsilon \in (0.05, 0.45)$, prior to the synchronization threshold ϵ_{th}, the coupled ratchets exhibits on-off intermittent synchronization [18]. A basic illustration of on-off intermittency is given in Fig. 3(a) for $\epsilon = 0.065$. The corresponding trajectories of the state space x_1(red) and x_2(green) exhibiting negative transport are shown in Fig. 3(b). Clearly, we see short-lived periods of constant signal ($\Delta x(t) = x_1(t) - x_2(t) \approx 0$) interrupted by larger amplitude bursts away from the synchronization manifold ($\Delta x(t) = 0$). We also see the on-off synchronized transport in the positive direction in Fig. 3(b) for $\epsilon = 0.45$.

An interesting mechanism by which the particles achieve OOI is also evident from Fig. 3(b). Here, we zoom a portion of Fig. 3(b) in the time interval $300 \leq t \leq 800$ as displayed in Fig. 3(c). During the earlier dynamics ($t \leq 400$) in Fig. 3(c), the particles advance in asynchronized (an on-state) manner and then get captured in the ratchet potential well, where they achieve temporal synchrony (off-state) - moving as a single entity and transporting periodic current. Due to the enhanced transport arising from synchronization, the particles acquire sufficient energy and suddenly, escape from the well, each particle again oscillating chaotically with larger amplitute in an asynchronized manner. During the on-state, the two particles slides over each other and in the off-state, move with the same displacement along the same path that is inevitably short-lived and then followed by unlocking, and so on. Thus, the repeated sequence of this motion (in forward or backward directions) gives rise to the OOI synchronized transport and clearly demonstrates the dynamical behavior of a stepwise-sliding motion of the actin-myosin system (that represents the most prominent group of multimotors in muscles). It has been reported that the contraction of muscle cells is a consequence of the sliding between thick filaments (actin) and thin filaments (myosin) aligned in parallel [20]. By tracking the course of sliding, it is possible experimentally to determine the size of the steps with nanometer precision (see [21–24] and references therein). This sliding could occur stepwise, in either forward or backward (corresponding to sarcomere lengthening) directions, or sometimes a combination of these steps; each step being $\approx 2.7nm$ and sometimes at inter multiples of $2.7nm$, which corresponds to the size of an actin monomer in the actin filament [21] or to the kinesin-microtubule system.

A natural question of interest is whether the intermittent transport of each particle in the OOI synchronized states differ from that of the collective dynamics (i.e. $\Delta x(t) = 0$). To confirm this, we characterize quantitatively the corresponding dynamics for $\epsilon = 0.065$ by computing the probability distribution of the laminar phase $\Lambda(t)$ of the difference variable $\Delta x(t)$ and the average laminar length of the variables $x_{1,2}(t)$ at the critical bifurcation point $a_c = 0.0809474$ as shown in Fig. 4. The collective dynamics $\Delta x(t) = 0$ of the coupled ratchets displays on-off intermittency satisfying a $-3/2$ power law scaling (Fig. 4(a)), while the individual dynamics exhibit type I intermittency [19] with a $-1/2$ power law scaling (Fig. 4(b)).

In order to understand in detail the accompaning dynamics and the transition to full synchrony, we examine the dynamics of the coupled system (1) for different val-

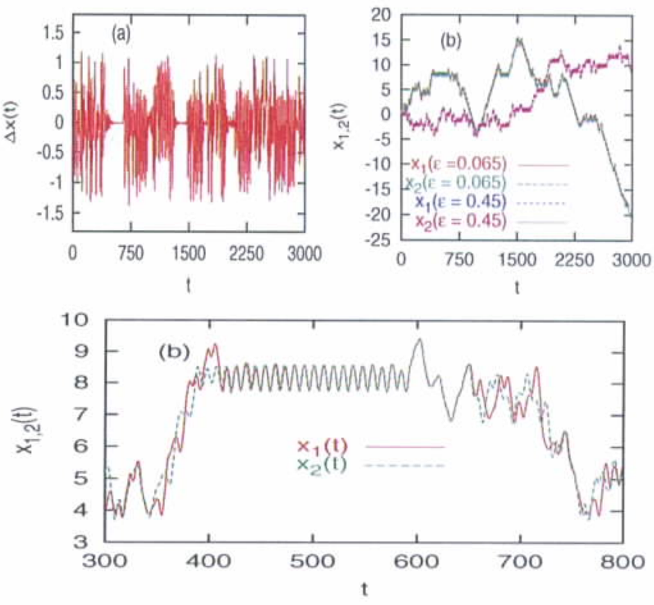

FIG. 3: Synchronization dynamics for the intermittent transport region. (a) On-off intermittency of the error state $\Delta x(t) = x_1(t) - x_2(t)$ for $\epsilon = 0.065$, (b) Directed intermittent chaotic transports for $\epsilon = 0.065$ (negative transport) and $\epsilon = 0.45$ (positive transport) and (c) Stepwise-sliding dynamics during an intermittent synchronized state for $\epsilon = 0.065$.

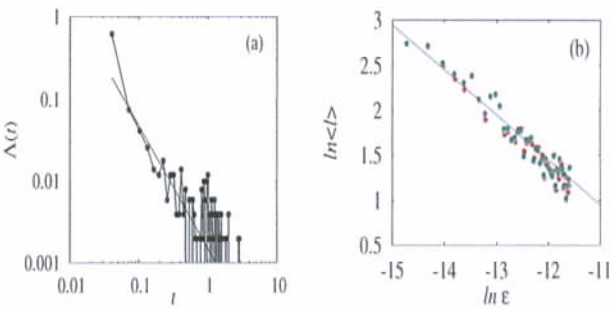

FIG. 4: Distribution of (a) laminar phases $\Lambda(t)$ of the difference $\Delta x(t)$ satisfying a $-3/2$ power law scaling typical of on-off intermittency and (b) average laminar lengths with varying parameter a satisfying the scaling law $< l > \ \propto \epsilon^{-0.5}$ with $\epsilon = a_c - a$ and $a_c = 0.0809474$ ($x_1(t)(red)$ and $x_2(t)(green)$). The coupling strength is $\epsilon = 0.065$.

ues of ϵ. For $\epsilon = 0$ (fixing other parameters) only one chaotic attractor exists in each of the subspaces. As ϵ is increased, the two ratchets begin to interact and the interaction leads to an attractor bubbling in which unstable periodic orbits co-exist with the chaotic attractors in both sub-spaces. Specifically, for ϵ near the on-off intermittency, the asymmetric period 4 attractor coexists with the chaotic attractor (Fig. 5(a)); these subsequently collides with the boundaries of the chaotic attractor and are destroyed in a crisis event that results in

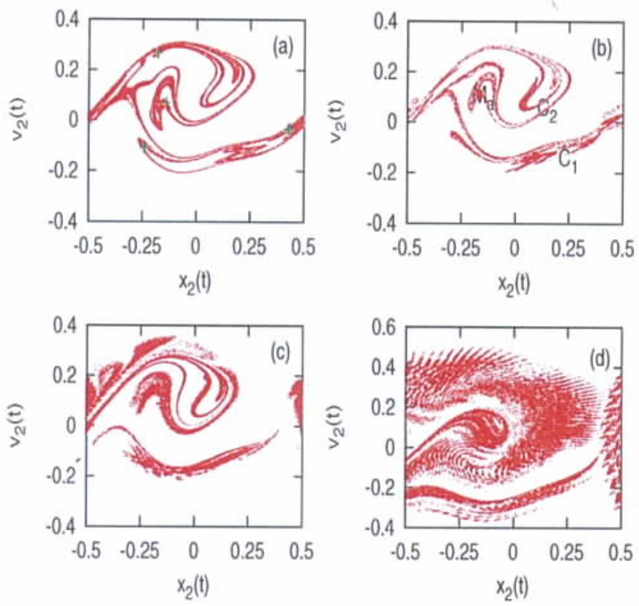

FIG. 5: Sequence of the transition in the subspace $(x_2(t), v_2(t))$ accompanied by crisis event. $v_2(t) = \dot{x}_2(t)$. (a) Co-existing chaotic and a period-4 attractor for $\epsilon = 0.002$, (b) co-existing chaotic attractors for $\epsilon = 0.0685$ just after the OOI at $\epsilon = 0.065$, (c) attractor splitting for $\epsilon = 0.25$ and (d) new chaotic attractor born during a synchronized state ($\epsilon = 0.9$).

the gradual collapse of the chaotic attractor (Fig. 5(b)). From the transition sequence shown in Fig. 5(b)-(d), the main body of the chaotic attractor (denoted by M_a) is weakly registered and gradually being split into two distinct chaotic attractors (denoted by C_1 and C_2). Finally, a new chaotic attractor that occupies nearly the entire phase space is born in the fully synchronized state (Fig. 5(d)), a signature of the enhanced transport resulting from the collective behavior of the coupled inertia ratchets (1).

We have shown in this letter that two coupled inertia ratchets achieve full synchrony via on-off intermittent synchronized state associated with crisis event during which each particle exhibits intermittent stepwise-sliding dynamics. This kind of motion does not occur in 1D ratchet models and provides a theoretical explanation of the origin of stepwise-sliding in molecular motors. The observation of stepwise-sliding in coupled ratchets and its connection with OOI synchrony, which has not been reported previously, further suggests that the step-size paradigm and its determination that consititute the main goal of a large number of recent experimental investigation on interacting molecular motors could be unravelled by observing synchronized dynamics. We also identified type I intermittency for each particle during the on-off intermittent state; and show that the sequence of transition from on-off intermittency to full synchrony leads to enhanced transport.

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