

Enumerating partially directed paths in a symmetric wedge

Abstract

The enumeration of lattice paths in wedges poses unique mathematical challenges. These models are not translationally invariant, and the absence of this symmetry complicates both the derivation of a functional recurrence for the generating function, and its solution. In this paper we consider a model of partially directed walks from the origin in the square lattice confined to a symmetric wedge defined by $Y = \pm X$.

We derive a functional equation for the generating function of the model, and obtain an explicit solution using a variation of the kernel method. The generating function turns out to be similar in form to Jacobi $\theta$-functions, and has a natural boundary on the unit circle.