Metriplectic Structure, Leibniz Dynamics and Dissipative Systems

Abstract:
A metriplectic (or Leibniz) structure on a smooth manifold is a pair of skew-symmetric Poisson tensor $P$ and symmetric metric tensor $g$. The dynamical system defined by the metriplectic structure can be expressed in terms of Leibniz bracket. This structure is used to model the geometry of the dissipative systems. The dynamics of purely dissipative systems are defined by the geometry induced on a phase space via a metric tensor. The notion of Leibniz brackets is extendable to infinite-dimensional spaces. We study metriplectic structure compatible with the Euler-Poincaré framework of the Burgers and Whitham-Burgers equations. This means metriplectic structure can be constructed via Euler-Poincaré formalism. We also study the Euler-Poincaré frame work of the Holm-Staley equation, and this exhibits different type of metriplectic structure. Finally we study the 2D Navier-Stokes using metriplectic techniques.